# Notes and Correspondence
# On an analytical model for the rapid intensification of tropical cyclones

Michael T. Montgomery[a,b]* and Roger K. Smith[c]
[a]Dept. of Meteorology, Naval Postgraduate School, Monterey, California, USA
[b]Hurricane Research Division, NOAA, Miami, Florida, USA
[c]Meteorological Institute, University of Munich, Germany
*Correspondence to: Michael T. Montgomery, Dept. of Meteorology, Naval Postgraduate School, Monterey, CA, USA.
E-mail: mtmontgo@nps.edu
The contribution of Michael T. Montgomery to this article was prepared as part of his official duties as a United States Federal Government employeee.

**Stimulated by recent developments in understanding tropical cyclones, we offer an evaluation of an analytical model that has been proposed to explain the rapid intensification of these storms. We articulate a number of concerns with this model, including the neglect of both the vertical momentum equation and the thermodynamic equation, and conclude that it falls a little short of achieving its stated aims. Copyright © 2010 Royal Meteorological Society**

*Key Words:* vortex dynamics; vertical momentum equation; thermodynamic equation

*Received 27 July 2009; Revised 20 October 2009; Accepted 2 December 2009; Published online in Wiley InterScience 10 February 2010*

*Citation:* Montgomery MT, Smith RK. 2010. On an analytical model for the rapid intensification of tropical cyclones. *Q. J. R. Meteorol. Soc.* **136**: 549–551. DOI:10.1002/qj.573

## 1. Introduction

In a recent paper, Kieu and Zhang (2009, henceforth KZ09) describe an axisymmetric analytical model that purports to examine the effects of organized deep convection on the rapid intensification of tropical cyclones. Deep convection in the model is represented by specifying a radially uniform distribution of vertical velocity everywhere within a prescribed radius of the rotation axis. This vertical velocity has a sinusoidal distribution in the vertical and is *an exponentially growing function of time*. Moist thermodynamical effects, including surface latent heat fluxes, which are known to be of crucial importance to intensification (Ooyama, 1969; Emanuel, 1986; Rotunno and Emanuel, 1987; Nguyen *et al.*, 2008; Montgomery *et al.*, 2009), are not accounted for explicitly. Friction effects are represented by a linear surface drag law that is presumed to be small enough such that these effects can be captured by a regular perturbation procedure in the drag coefficient. The authors suggest that the separable solution they obtain is useful for interpreting observed rapid intensification phases of tropical cyclones as well as those simulated by recent full physics atmospheric numerical models.

The main conclusions offered are (in KZ09's words) that '(1) the rotational flows in the inner-core region grow double-exponentially, and the central pressure drops occur at rates much faster than the rotational growth; (2) the amplification rates of the primary circulations differ profoundly from those of the secondary circulations; (3) the rotational flows tend to grow from the bottom upwards with the fastest growth occurring at the lowest levels; and (4) the tropical-cyclone growth rates depend critically on the vertical structure of tangential flows, with a faster rate for a lower-level peak rotation.' Moreover, 'the central pressure drops (*sic*) could occur at the squared double exponential rates'. KZ09 advocate the adoption of this solution as a way of improving the initialization of an intensifying tropical-cyclone vortex.

## 2. Critique

Neither the physical reasons for the foregoing results, nor their relevance to understanding tropical cyclones are

| Report Documentation Page | Form Approved OMB No. 0704-0188 |
|---|---|

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

| 1. REPORT DATE | 2. REPORT TYPE | 3. DATES COVERED |
|---|---|---|
| **OCT 2009** | | **00-00-2009 to 00-00-2009** |

| | |
|---|---|
| 4. TITLE AND SUBTITLE<br>**On an analytical model for the rapid intensification of tropical cyclones** | 5a. CONTRACT NUMBER<br>5b. GRANT NUMBER<br>5c. PROGRAM ELEMENT NUMBER |
| 6. AUTHOR(S) | 5d. PROJECT NUMBER<br>5e. TASK NUMBER<br>5f. WORK UNIT NUMBER |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)<br>**Naval Postgraduate School,Department of Meteorology,Monterey,CA,93943** | 8. PERFORMING ORGANIZATION REPORT NUMBER |
| 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) | 10. SPONSOR/MONITOR'S ACRONYM(S)<br>11. SPONSOR/MONITOR'S REPORT NUMBER(S) |

12. DISTRIBUTION/AVAILABILITY STATEMENT

**Approved for public release; distribution unlimited**

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

| 16. SECURITY CLASSIFICATION OF: | | | 17. LIMITATION OF ABSTRACT | 18. NUMBER OF PAGES | 19a. NAME OF RESPONSIBLE PERSON |
|---|---|---|---|---|---|
| a. REPORT<br>**unclassified** | b. ABSTRACT<br>**unclassified** | c. THIS PAGE<br>**unclassified** | **Same as Report (SAR)** | **3** | |

clear to us. Moreover, while we are very sympathetic to KZ09's desire to develop a simplified understanding of the rapid intensification of a tropical cyclone, we have serious reservations with their approach, as well as with the mathematical and physical consistency of their formulation. In the light of our recent findings on the dynamics of tropical cyclones, we think it is worth exposing the more problematic of these concerns as itemized below.

- We know of no observational evidence to support the assumption that the mean upward motion in regions of deep convection increases *exponentially* with time. KZ09 present a figure in support of this claim, based on an areal average of vertical velocity over a 400 km radius in a numerical simulation of hurricane *Wilma* (2005). Unfortunately, the axes on this figure have no units and the ordinate scale is linear rather than logarithmic, making it difficult to judge exponential as opposed to algebraic growth. Even so, for typical-sized tropical cyclones, such a large area would extend into the region of persistent subsidence. Physical considerations would suggest that the maximum vertical velocity in the convective cores of developing hurricanes should decrease with time (Ooyama, 1969). Recent cloud-representing numerical model simulations indicate that convective updraughts become less intense because of the stabilizing effect of the upper-level warming that accompanies the intensification process and also because of the increasing inertial stiffness of their environment (Hidalgo, 2007; Nguyen *et al.*, 2008). This result is consistent with lightning observations of Price *et al.*(2009), who showed that peak lightning activity occurs typically one day prior to the first peak in maximum wind speed (their Figure 3). In citing these observations, we recall that lightning activity is favoured by convective updraughts greater than approximately $10\,\mathrm{m\,s^{-1}}$. Since both the modelling and observational evidence suggests that the number of updraughts exceeding this threshold decreases with intensity, it seems implausible to us that the areal average of vertical velocity will increase exponentially. Even so, irrespective of this assumed growth in vertical motion, a key requirement of a theory of rapid intensification should be that it *predicts* the rate of intensification rather than imposing it, or at least one component of it.
- A non-negligible fraction of the pressure fall at the centre of a tropical cyclone is attributed to the warmth of the eye, which in turn is associated with subsidence during the intensification phase (Malkus, 1958; Willoughby, 1998). Not only does the prescription of ascent throughout the entire core region preclude the subsidence warming in the present formulation, it must lead to adiabatic cooling. If unabated, this cooling will generate negative system buoyancy[†] thereby opposing the prescribed vertical velocity and if this air is advected radially outwards in the upper troposphere it must lead to dry convective instability. The authors appear not to notice this physical inconsistency. In reality, the cooling of the ascending air will be partially offset by latent heat release, which KZ09 do not represent in their formulation: the thermodynamic equation (their Eq. (6)) is not used subsequently in their analytical solution.
- The authors use the continuity equation to determine the time evolution of the radial velocity, from which they infer both the evolution of the tangential wind field *and* the radial variation of pressure, or geopotential. The important constraints implied by the thermodynamic equation and vertical momentum equation are not accounted for and it would be fortuitous, indeed, if these equations were satisfied. This neglect may explain why the authors failed to notice the physical inconsistency discussed in the last item. We caution that ignoring one or more components of Newton's second law can lead to false dynamical understanding, as highlighted in assuming gradient wind balance in the tropical-cyclone boundary layer without a rigorous justification (Smith *et al.*, 2008; Smith and Montgomery, 2008). Slaving to the radial pressure gradient to the radial velocity, which, itself, is determined kinematically by the prescribed vertical motion, is counter to the spirit of Newton's second law. It is inconsistent also with conventional swirling boundary-layer theory, in which the radial flow is intimately related to the frictional disruption of gradient wind balance (Smith, 1968; Eliassen and Lystad, 1977; Kepert, 2001; Montgomery *et al.*, 2001; Smith and Vogl, 2008; Smith *et al.*, 2009; Bui *et al.*, 2009).

While we have focussed above on the physical attributes of KZ09's model, one can summarize the solution steps from a mathematical perspective. Basically, the prescription of vertical velocity allows one to solve for the radial motion using the continuity equation assuming axial symmetry. One then obtains the tangential velocity from the tangential momentum equation and the pressure distribution by radially integrating the radial momentum equation. KZ09 terminated their solution at this point. In order to obtain a complete solution, two further steps are necessary and for simplicity we illustrate these in the context of the hydrostatic and Boussineq approximations. First, the space–time distribution of system buoyancy is obtained from the vertical derivative of the pressure. Then, the thermodynamic equation yields the heating that is required to produce the changes in the buoyancy distribution. Of course these two steps are not as simple when one relaxes either the hydrostatic approximation or the Boussinesq approximation. Regardless, KZ09 did not present solutions for buoyancy and heating, and it would be fortuitous if these were realistic. Nevertheless, the basic problem remains that there is no plausible chain of causality that would lead to an arbitrary prescription of the vertical velocity distribution in the first place.

## 3. Conclusion

We recognize the importance of understanding the rapid intensification of tropical cyclones and are sympathetic to the idea of developing simple models that elucidate the underlying dynamical and thermodynamical processes.

[†]The term *system buoyancy* was discussed by Smith *et al.*(2005) and refers to buoyancy defined relative to an *ambient* reference density that depends only on the height above the surface.

 

However, for the reasons discussed above, we question the validity of the time-dependent solution presented by KZ09 as well as its usefulness in understanding rapid intensification.

### Acknowledgements

This paper was written during the authors' attendance at the IAMAS/IAPSO MOCA conference in Montreal in July 2009. We thank Kerry Emanuel for his review of our manuscript and for encouraging us to include the summary of mathematical steps. MTM acknowledges the support of grant no. N00014-03-1-0185 from the US Office of Naval Research and from the US Naval Postgraduate School, NOAA's Hurricane Research Division, and NSF ATM-0649944, ATM-0649946 and ATM-0715426. RKS acknowledges financial support for hurricane research from the German Research Council (Deutsche Forschungsgemeinschaft).

### References

Bui HH, Smith RK, Montgomery MT, Peng J. 2009. Balanced and unbalanced aspects of tropical-cyclone intensification. *Q. J. R. Meteorol. Soc.* **135**: 1715–1731.

Eliassen A, Lystad M. 1977. The Ekman layer of a circular vortex: A numerical and theoretical study. *Geophys. Norv.* **31**: 1–16.

Emanuel KA. 1986. An air-sea interaction theory for tropical cyclones. Part I: Steady state maintenance. *J. Atmos. Sci.* **43**: 585–604.

Hidalgo JM. 2007. 'Vortical hot towers, their aggregate effects, and their resolution dependence in the formation of tropical storm *Diana* (1984)'. PhD thesis, Colorado State University, USA.

Kieu CQ, Zhang D-L. 2009. An analytical model for the rapid intensification of tropical cyclones. *Q. J. R. Meteorol. Soc.* **135**: 1336–1339.

Kepert JD. 2001. The dynamics of boundary layer jets within the tropical cyclone core. Part I: Linear theory. *J. Atmos. Sci.* **58**: 2469–2484.

Malkus JS. 1958. On the structure and maintenance of the mature hurricane eye. *J. Meteorol.* **15**: 337–349.

Montgomery MT, Snell HD, Yang Z. 2001. Axisymmetric spindown dynamics of hurricane-like vortices. *J. Atmos. Sci.* **58**: 421–435.

Montgomery MT, Nguyen SV, Smith RK, Persing J. 2009. Do tropical cyclones intensify by WISHE? *Q. J. R. Meteorol. Soc.* **135**: 1697–1714.

Nguyen SV, Smith RK, Montgomery MT. 2008. Tropical-cyclone intensification and predictability in three dimensions. *Q. J. R. Meteorol. Soc.* **134**: 563–582.

Ooyama KV. 1969. Numerical simulation of the life cycle of tropical cyclones. *J. Atmos. Sci.* **26**: 3–40.

Price C, Asfur M, Yair Y. 2009. Maximum hurricane intensity preceded by increase in lightning frequency. *Nature Geosci.* **2**: 329–332.

Rotunno R, Emanuel KA. 1987. An air–sea interaction theory for tropical cyclones. Part II: Evolutionary study using a non-hydrostatic axisymmetric numerical model. *J. Atmos. Sci.* **44**: 542–561.

Smith RK. 1968. The surface boundary layer of a hurricane. *Tellus* **20**: 473–484.

Smith RK, Montgomery MT. 2008. Balanced boundary layers in hurricane models. *Q. J. R. Meteorol. Soc.* **134**: 1385–1395.

Smith RK, Vogl S. 2008. A simple model of the hurricane boundary layer revisited. *Q. J. R. Meteorol. Soc.* **134**: 337–351.

Smith RK, Montgomery MT, Zhu H. 2005. Buoyancy in tropical cyclones and other rapidly rotating atmospheric vortices. *Dyn. Atmos. Oceans* **40**: 189–208.

Smith RK, Montgomery MT, Vogl S. 2008. A critique of Emanuel's hurricane model and potential intensity theory. *Q. J. R. Meteorol. Soc.* **134**: 551–561.

Smith RK, Montgomery MT, Nguyen SV. 2009. Tropical cyclone spin-up revisited. *Q. J. R. Meteorol. Soc.* **135**: 1321–1335.

Willoughby H. 1998. Tropical cyclone eye thermodynamics. *Mon. Weather Rev.* **126**: 3053–3067.

 